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Thinking and Being are not the Same¹

This paper is part of a larger project, one aim of which is to inquire critically into the forms of thought, language and life that become possible outside the constituted tradition of Western metaphysics and its determining inscription of the One as ultimate cause.² The critical inquiry is positively directed toward developing what I call several "logics of ineffectivity." These are logics that witness -- by contrast with this constituted tradition and the efficacious symmetry or identity of thought and being it presupposes - the ultimate *incapacity* of thought consistently to determine beings as such and as a whole.

A central part of this inquiry is to consider critically the rational ideas that have offered justification to the *human subject* of judgment, agency, or action in its claim to dominance, in its thought and action, over non-human beings in general and as such. In this paper, I shall consider, specifically, a certain historically prominent and recently influential idea of the *general rational capacity of judgment*. This idea plays an essential role in some prominent recent projects within analytic philosophy (such as those of Irad Kimhi, Sebastian Rödl, and John McDowell), that attempt to rehabilitate, in whole or part, a project of absolute idealism like that associated with Hegel in his overcoming of Kant. But it is originally rooted in Aristotle, who understands it as the idea of the capacity (*dunamis*) *meta lo*gou and describes its structure in *Metaphysics*, book IV (Θ), chapter 2. Before this, some main aspects of the idea are anticipated by Plato in the form of the Eleatic Stranger's account of knowledge and judgment in the *Sophist*.

According to the relevant idea, the *capacity for judgment* is one that is distinctively possessed by human beings as something present in the soul, self, or subject of such an animal. It endows its possessor with the ability, in general, to make a "two-way" exercise of judgment with respect to any proposition or propositional content *p* within the scope of its powers of knowledge: that is, it enables its bearer either to judge *p*, or *not-p* as true, on the basis of inference and the evidence available to it. Within the scope of this idea, the unity of logical structure between thought and the world is captured by the claim that thinking and being are the same, in this sense: that what *can* be thought – what it is possible for a thinker to think, in the sense of determining judgment – has the *same form* as that which is, or can be, the case. A further commitment of the relevant picture of capacities, only implicit in Plato and Aristotle but central to its modern development, is that exercises of the capacity are exercises of *self-conscious* reflexivity: that is, that in engaging the capacity to arrive at the judgment that *p* or the judgment that *not-p*, I am necessarily *aware of myself* as doing so.

The idea of a capacity for rational judgment *in general* is the idea of a capacity that, in itself, has no limits as to its subject matter or domain. It is the idea of a power that, as it can relevantly be brought to bear on any subject matter or topic whatsoever, is itself separate from any positive predicative determination or description: separate, that is, from any determination as to what *it*, itself is. As it is separable in this way, it is not determinable as a natural being. Accordingly, if there is such a capacity,

¹ This paper was presented at the 43rd annual ALWS symposium in Kirchberg in August, 2022, and before that (in a rather different form, and via zoom) as a University of New Mexico philosophy department colloquium in March, 2021. I would like to thank the participants at both discussions for helpful comments and questions.

² Cause, that is, both of being and of thinking: cf. Plato, *Republic*, 508d-509b.

its possession suffices to distinguish its possessor from any part or aspect of the material or natural world: indeed from any aspect of the world itself, understood as comprising *all that is the case*. Having this generality, it is then –according to the relevant idea -- fundamentally unlike, because fundamentally broader than, the more *limited* powers of perception that, by contrast, we plausibly share with non-human animals.³

But as I shall argue here, there *cannot be* any such general rational capacity that is *actually possessed* by such thinkers as we are, and know ourselves to be. For its idea, as I shall argue, is the idea of a form of self-consistent general unity that is at the same time a consistent unity of thinking, and of being, and of the two (thinking and being) with respect to each other. But as I shall argue, this idea of twofold consistency can no longer be maintained, once we see the relevant activities of judgment as *embodied in the structure of a natural language* and *carried out by the finite speakers or users of such a language*. Since the idea of consistency that underlies it cannot be maintained, the idea of a *general rational capacity of judgment* possessed by these speakers or users, and marked by this unitary form of consistency, must be rejected as well. With this, one aspect of a historically decisive and still dominant idea of the distinctive entitlement of the human is shown to be illusory; and one characteristic claim to power of the human subject of thought and decision, accordingly, can be allowed to lapse.

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Familiarly, judgment presents itself as inherently *general*. That is, when I judge *that p* on the basis of whatever evidence (perceptual or rational) supports that judgment, my activity in judging instances a more general *type* of activity that appears, in itself and as such, not to be pre-determined as to any specific *content* or *object*. My activity seems to be general, that is, in that another instance of what is recognizably the same kind of activity could be operative in another judgment about quite a different subject matter, or with respect to quite different objects. At the same time, the generality of what allows me to judge *that p* for arbitrary *p* appears closely connected to the possibility of *negation*: in judging that *p* – that *p* is the case or that "p" is true– I also judge that it is *not the case* that ~*p*, or that "not: p" is not true. This generality can be expressed by saying that, as a matter of the generality of the activity of judgment just in itself, it appears possible to judge *p* true, or false, for *any* propositionally structured "content" *p* that *is* true or false and that I understand. As judging that *p is false* is judging that *it is not the case that p*, this generality is the same as the generality of negation: that is, as the generality of the possibility that appears in judging *what is not the case*.⁴

Since Parmenides, it has appeared mysterious that we can make "negative" judgements, as there is apparently *nothing* in being for such a judgment to correspond to. Since (at least) Plato's argument in the *Sophist*, forms of response to Parmenides' problem have invoked a systematic idea of the generality of what we may understand as a *logic* of judgment: that is, a systematic and general relationship of the

³ For this argument, see Aristotle, *On the Soul* III.4-6, especially III-4, 429a18-28.

⁴ This characteristic generality of thought can be put, in a way that is not specific to judgment, as a matter of thought's adhering to what Evans (1982, pp. 100-103) calls the "Generality Constraint." According to the constraint, someone who can be credited with the thought that *a* is *F* must possess the "conceptual resources" for entertaining the thought that *a* is G for every property G of which they have a conception (and, *a fortiori*, of *a's* not being-G or its being non-G). (I owe thanks to Hilan Bensusan for suggesting this way of putting the matter).

logical forms of possible judgment to the forms of what is, or possibly *can* be, the case.⁵ The idea of such a systematic logic is the idea of a structure of possibility that is *indifferently* the structure of what *can* be thought or what *can* be (although it indeed *may* not be) the case: it is the idea, in other words, that there is, on the level of ultimate logical form or structure, no distinction between the one and the other, between what *can* be thought and what *can* be the case.

In the *Philosophical Investigations,* section 95, in the context of an explicit critical dialogue with his own earlier position in the *Tractatus,*⁶ Wittgenstein stages in characteristic form one of the core ideas underlying the suggestion of such a logic, as well as one kind of problem ("paradox") to which it has attempted to respond:

95. "Thinking must be something unique." When we say, *mean*, that such-and-such is the case, then, with what we mean, we do not stop anywhere short of the fact, but mean *such-and such* – *is* – *thus-and-so*. -- But this paradox (which indeed has the form of a truism) can also be expressed in this way: one can *think* what is not the case.⁷

In Mind and World, McDowell suggests that at least the "truistic" aspect of Wittgenstein's remark can be read as suggesting a kind of overall formal identity between what is thought when one thinks truly, and the world itself. On this conception, although thought can be "distanced from the world" by being false, there is no distance from the world that is involved in the "very idea" of thought itself: no (as McDowell also puts it) "ontological gap" between what one can think and "the sort of thing that can be the case."8 Conceptual thinking can assuredly be, on this view, constrained from outside its own activity, and indeed perception evidences one kind of such constraint. But although conceptual thinking can thus be constrained by what is outside it, it is not constrained from outside the space of what is *thinkable* (what McDowell calls the "logical space of reasons"), which indeed has no such external constraint or bound with respect to the totality of facts itself.⁹ Rather, according to a picture that McDowell says "can be reworked ... for any conceptual shaping of subjectivity," the conceptual capacities that must be credited to any subject of (say) genuine perceptual experiences will be capacities whose exercise is, as such, always already a placing of contents within the "logical space of reasons," a bringing "into view" of what is a reason for what.¹⁰ It is then the subject's self-conscious responsiveness to these rational relations among possible contents of judgment in general that qualifies this placing to be a matter, not of mere responses or reactions, but actual takings of position with respect to how the world is, or is not.¹¹

As Sebastian Rödl argues in a recent (heterodox) internalist defence of the *objectivity* of knowledge as issuing from judgment in this sense, the power underlying it, being characteristically *general*, is not determined as a power *to* (do) something, as opposed to anything else. Instead:

⁵ Kant's deduction, in the *Critique of Pure Reason*, of the categories from the possible forms of judgment is, of course, another instance of this kind of systematic logic.

⁶ See the immediately following section 96, as well as the explicit citation and quotation of the *Tractatus* in sections 97-98.

⁷ Translation slightly modified.

⁸ McDowell (1996), p. 27.

⁹ McDowell (1996), p. 28.

¹⁰ McDowell (1996), p. 29.

¹¹ McDowell (1996), p. 30.

...in judging, I understand my judgment to spring from a power which, in its ultimate description, is the power of knowledge. Indeed, I so comprehend anyone's judgment ... Since judgment is objective, the power of knowledge is not a power to this or that; it is *the* power, the power *überhaupt*.¹²

Such a power is not one, Rödl suggests, that is "as it is anyway," (as, for example, sensory powers by contrast are "as they are anyway") quite independently of what is revealed in its acts.¹³ Rather, given its generality which is the generality of *all* objective knowledge, it is to be understood only as it determines itself in and through these acts themselves. In this articulating, according to Rödl, the power of knowledge comes to know itself, and in this self-articulation we find knowledge of (among other things) "the principles of judgment" in which "thought and being are known to be the same."¹⁴ The first and most general among them, according to Rödl, is the law of non-contradiction, which I am thus aware of my use of in *any* judgment that admits of a contrary.¹⁵

In a recent and similarly radical investigation, Irad Kimhi argues that a defense of the sameness of thinking and being – and thus an articulation of "philosophical logic," in one sense of that term – can proceed by way of explicating the "truistic" claim that McDowell also draws from Wittgenstein's remark at PI 95.¹⁶ According to Kimhi, a removal of the "misunderstandings" that keep us from seeing this sameness, can provide the basis for a correlative recognition and acknowledgement of what underlies the "astonishment" of the remark's first (quoted) sentence: an astonishment at the "uniqueness" of thought in its sameness with being, at its being (in that respect) *unlike* anything else.¹⁷ Like Rödl and McDowell, Kimhi proceeds by elucidating a capacity of self-conscious judgment that is understood as being characteristically *general* as to scope and content.¹⁸ As the explication of such a capacity is the explication of a capacity that bears the generality of all that can be thought -- a generality that is *also* that of all that can be the case, Kimhi argues that it will succeed only if it can succeeds in vindicating what he terms the "syllogisms of thinking and being."

¹⁷ Although my primary aim here is not Wittgenstein exegesis, it is to be noted that both McDowell's and Kimhi's exegeses of *PI* 95 pay little attention to its textual context and in fact seem quite opposed to Wittgenstein's own methodological intentions there. In context, the statement in quotation marks at the beginning of the remark is not the expression of an "astonishment" that Wittgenstein himself shares but rather an expression in an interlocutory voice of a characteristic tendency to "sublimate" (PI 94) the logic of our language, a tendency that arises from a "misunderstanding", one that "sends us in pursuit of chimeras" arising from our inability "simply to look and see how propositions work" (PI 93-94). In this context, it also appears evident that Wittgenstein's main intention in staging the "truistic" or "paradoxical" thought that one can think what is, or what is not, the case, is not to point to a "deep" philosophical problem to be confronted on its own terms, but rather to point to the way in which that sense of there being a "deep" problem is itself grounded in a tendency to be misled by the forms of our language: especially, of the ways we express ourselves using general terms such as "being" and "thought." (Compare the somewhat parallel discussion that frames Wittgenstein's treatment of the question of the possibility of "thinking what is not the case" in the *Blue Book* (Wittgenstein 1958, pp. 30-31), where just this question is said to be a "beautiful example of a philosophical question" in that it arises only "when we look at the facts through a misleading form of expression.")

¹⁸ Kimhi (2018), p. 16.

¹² Rödl (2018), p. 17.

¹³ Rödl (2018), p. 17.

¹⁴ Rödl (2018), p. 17.

¹⁵ Rödl (2018), pp. 139-40.

¹⁶ Kimhi (2018), pp. 12-13.

Two examples of such syllogisms are these (where "A" stands for a subject or agent and "p" stands for a declarative sentence):

- 1) A thinks p
- 2) not-p (p is not the case)
- 3) A falsely thinks p

And

- 1) A thinks not-p
- 2) not-p (p is not the case)
- 3) A truly thinks not-p¹⁹

As Kimhi points out, the availability of these syllogisms is obscured the usual assumption of the distinction between "extensional" and truth-functional or "intensional" and non-truth-functional contexts in which a sentence can appear.²⁰ For on this assumption, the occurrence of p (or not-p) in the first and third lines of either of the syllogisms must rather be an occurrence of "p", where the quotation marks indicate that what appears within them does not appear in the same (truth-functional) way that it appears in the second line. In order to defend them, it will thus be necessary, as Kimhi argues, to envisage a way in which a sentence can appear, within another, in a non-truth-functional but nevertheless still logically significant way.

As Kimhi further suggests following what is plausibly Aristotle's own conception, if the unity of thinking and being is to be vindicated in the context of a picture of a capacity of judgment that is *general* in the sense discussed, its defence will include a vindication of the principle of noncontradiction, as it applies *both* to thinking and to being. As is well known, Aristotle introduces the principle, calling it the "firmest of all", in at least two different forms. What can be called (following Kimhi) the *ontological* principle of noncontradiction affirms that it is impossible for the same thing to bear and not bear the same property at the same time and in the same respect. By contrast, the *psychological* principle of noncontradiction affirms that it is impossible for anyone to *believe* that the same thing is and is not.²¹ While the first principle appears to be a principle of being, placing a limitation on what can be or be the case, the second appears to be a principle of thinking, placing a limitation on what can be believed or thought.²² As Kimhi recognizes, then, upholding the identity of thinking and being will require discerning these two expressions as expressing what is recognizably the *same* principle in two different ways.

On the position of "psycho/logical monism" that Kimhi accordingly defends, the unity and identity of what is displayed by a propositional sign (across supposedly "intensional" and "extensional" contexts) is, accordingly, the unity and identity of a repeatable *act* of consciousness.²³ This unity will be the same

¹⁹ Kimhi (2018), p. 10. As Kimhi is aware, putting the syllogisms this way involves a (deliberate) violation of now-typical "conventions" of use and mention.

²⁰ Kimhi (2018), pp. 11-12.

²¹ Kimhi (2018), pp. 25-29.

²² Kimhi (2018), pp. 25-31. This distinction follows Lukasiewicz, who also (however) distinguishes a third, "logical" principle (at *Metaphysics* IV 6, 1011b13-14) according to which contradictory sentences *cannot be true* at the same time.

²³ Kimhi (2018), pp. 52-53.

wherever the same propositional sign appears, and thus, *a fortiori*, will be the same in *p*, $\sim p$, and *I think p*. And since, on Kimhi's view, the capacities drawn on in producing all three are the same, the capacity whose structure underlies both the ontological and psychological principles of noncontradiction will be the same as well. Possessing it will then involve, at minimum, an ability to deploy it in truth-functional contexts such as that of negation *as well as* reflexively intentional ones such as "I think...". From this point of view, Kimhi argues, as bearers of such a capacity we come to see that no conscious act – and therefore no possible judgment – is displayed by something of the form *p* and $\sim p$ and, accordingly, that neither "*p* and *not-p*" nor "*not: p* and *not-p*" are genuine propositions, not genuine expressions of anything I can think *or* anything that can be the case. Seeing this, we understand the principle of noncontradiction, in such a way that it bears with generality on both thinking and being: we see, in other words, that the same principled unity that applies to the appearances of *p* in thought in general *also* applies to its unity in being.²⁴

Within the context of a philosophical sensibility characteristic of some of the historical philosophers Kimhi, Rödl, and McDowell draw on, the activity of thinking is, as such, often conceived as an essentially *interior* act of a self-conscious subjectivity. Within the context of such a sensibility, it can seem obvious that we, in our thinking about the world, must *ourselves* be thinkers in the (idealized) sense this suggests. But a recognizably different philosophical sensibility – one that may be identified with Wittgenstein as opposed to (say) Aristotle or Kant -- is indicated by giving prominence to the fact that our activities of thinking are, where we can explicate them to ourselves, essentially understandable as activities of operating and acting with language.²⁵ By foregrounding this use, we bring what we might call the life of language – that is, *our* life *with* language – to bear on the question of the significance of this language as it offers us terms (the only terms we have, from this perspective) to articulate that knowledge we can come to have of the possible forms of this life.²⁶ Here, of course, the grammar of "can" and of "capacity" is also not self-evident: it is not to be assumed at the outset that we know or have any adequate general conception of what it is for us to "have" a capacity or to "be able" in general to think, judge, say (etc.) *that* ...²⁷

So the question arises, in a context shaped by this kind of recognition of the role of language in our lives: is it even (so much as) coherent to suppose that there can be a *general* capacity of judgment that is possessed by the speakers and users of language that we thus know ourselves to be?

²⁴ Kimhi (2018), p. 55; p. 67.

²⁵ Kimhi, Rödl, and McDowell are, of course, not *blind* to this difference of philosophical sensibilities or methods. Indeed, all three take themselves to work within a methodological context definitively shaped by the recognition of the latter perspective (see, especially, Kimhi (2018), pp. 61-66). However, if the arguments to follow are successful, they all miss the extent to which taking this perspective requires calling into questions central and defining commitments of the earlier one (most directly, its commitment to the subject as the *locus* of the (supposedly unitary) activity of thought).

²⁶ Compare, e.g., Wittgenstein in the *Blue Book*: "As part of the system of language, we may say, the sentence has life. But one is tempted to imagine that which gives the sentence life as something in an occult sphere, accompanying the sentence. But whatever accompanied it would for us be just another sign …" (pp. 5-6) ²⁷ Cf. Wittgenstein's critical discussion of the grammars of "is", "can", and "able to" and their relation to the grammar of "know," as well as to questions of "mental processes" and the "ability" of understanding, that begins at *Pl* 149-150.

In the *Notes on Logic* that Wittgenstein dictated to Russell in 1913, he says that:

When we say that A judges that, etc., then we have to mention a whole proposition which A judges. It will not do to mention only its constituents, or its constituents and form but not in the proper order. This shows that a proposition itself must occur in the statement to the effect that it is judged; however, for instance, "not-p" may be explained, the question what is negated must have a meaning.28

And:

In *not-p*, p is exactly the same as if it stands alone; this point is absolutely fundamental.²⁹

Thus, combining the two remarks, Wittgenstein held already in 1913 that what is expressed by the propositional symbol p is the same in all of the following contexts: p (alone), ~p, and A judges that p. Somewhat famously, the conception expressed in the first remark – that p itself must appear in the context "A judges that p'' – played a central role in motivating Wittgenstein's decisive objection in June 1913 to the "multiple relation" theory of judgment that Russell was attempting to advance in his book *Theory of Knowledge*, leading Russell to give up working on the book.³⁰ One crux of the objection appears to have been – as Wittgenstein later says at Tractatus 5.5422 – that the correct theory of judgment (i.e. the correct theory of the form of the proposition "A judges that p") must show that it is impossible to judge a nonsense. Russell's theory, which saw the form of "A judges that p" as a matter of the multiple relations of A to the several components of p, did not satisfy this requirement, since it did not require that the objects of A's various relations exhibit the unity characteristic of genuine propositions with sense, the unity that allows p to stand (if it does) for a genuine proposition. In order for this unity to be exhibited in "A judges that p," it must be that p itself, with its characteristic unity, appears within the longer proposition: as Wittgenstein's 1913 remark underscores, no mere combination of constituents, or constituents along with a form, will do.

As Kimhi suggests, this conception centrally captures one aspect of the claim of a formal identity of thinking and being. Or more exactly – as we may put it in Wittgenstein's own terms – it captures one aspect of the formal identity between *language* (the totality of propositions, according to 4.001) and reality, the identity which Wittgenstein calls (in general terms) "logical form" (2.18). The logical form of a sign is a matter of its "logico-syntactical" employment (3.327) or use: it is only in the context of its use in a language (as a whole) that a sign, including a propositional sign, has and exhibits the unity it does. This includes the "contradictory unity" of p with $\sim p$, as well as (in connection in particular with "psychological" verbs such as "thinks" and "believes") of p as it appears in non-extensional contexts.

Now, what consequences does this unity have, in the context of a general account of capacity of language to picture reality, for the question of the *constitution* of a (or "the") subject itself? In the

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²⁸ Wittgenstein (1979), p. 94.

²⁹ Wittgenstein (1979), p. 95.

³⁰ As Michael Potter (2009) notes, exegesis of the episode has become "a sort of rite of passage for scholars of early analytic philosophy;" for some of these treatments, see Potter's references (2009, p. 119)

Tractatus at 5.54-5.5421, Wittgenstein develops an account of the structure of the general propositional form and the special form of "intentional" sentences of belief, thinking and saying:³¹

5.54 In the general propositional form propositions occur in other propositions only as bases of truth-operations.

5.541 At first sight it looks as if it were also possible for one proposition to occur in another in a different way.

Particularly with certain forms of proposition in psychology, such as 'A believes that p is the case' and 'A thinks p' ['A denkt p'], etc.

For if these are considered superficially, it looks as if the proposition *p* stood in some kind of relation to an object *A*.

(And in modern theory of knowledge (Russell, Moore, etc.) these propositions have actually been construed in this way.)

5.542 It is clear, however, that 'A believes that p', 'A thinks p', and 'A says p' are of the form ""p" says p': and this does not involve a correlation of a fact with an object, but rather the correlation of facts by means of the correlation of their objects.

5.5421 This shows too that there is no such thing as the soul – the subject, etc. – as it is conceived in the superficial psychology of the present day. [...dass die Seele – das Subjekt etc. ... ein Unding ist.]

Indeed a composite soul would no longer be a soul.

The passage moves rapidly from considerations about the mode of figuring of one proposition within another to the denial of the existence of a soul or subject "as it is conceived in the superficial psychology of the present day:" that is, of a soul or subject that would have the compositeness of a subject that thinks or judges now this, now that.³² The crucial consideration in producing this result is the "extensionalist" requirement of 5.54: that a sentence can appear within another sentence *only* truth-functionally. Given this, when a sentence appears within one of the forms of sentences usually used as expressions of belief, assertion, judgment, or thinking, it cannot be the case that the longer sentence affirms – as it superficially seems to – a *relationship* of a subject to a fact, proposition, or content. Rather, according to the argument of the passage, the superficial grammar according to which such an intentional sentence appears to relate a substantial subject, mind, or soul to a proposition is to be

³¹ It is significant that this is *not* simply a theory of "judgment," though, but rather a *general* theory of propositional intentionality.

³² This nonexistence of the thinking subject is confirmed elsewhere in the *Tractatus*; most notably at 5.631, in connection with the consideration of *The World as I found it*: "The thinking and representing subject does not exist" [*Das Denkende, Vorstellende Subjekt gibt es nicht.*] Following this, Wittgenstein underscores that the "metaphysical subject" [5.633] or "philosophical self" [5.641] is not "the human being," the "human body," or the "human soul," but rather a [or the] "limit of the world" [5.632; 5.641].

replaced with the disquotational sentence of the form Wittgenstein suggests: not "A says 'p'" but "'p' says p".³³

At first glance, this requirement of general extensionalism appears to contradict Wittgenstein's claim in the 1913 Notes: that in "A judges that p", p itself must appear; for it appears as if, in the latter, p appears (if it does at all) only in a non-truth-functional way. In fact, there is no contradiction. The key to seeing that the two are not in contradiction is to see that it is not the appearance of p in "A judges that p'' that is illusory, but rather the appearance of A – the (supposed) subject – itself. The unity of p is in *no* sense correlative to a unitary A; the full expression is not in any sense the expression of a "propositional attitude" taken by a subject toward a proposition. At the same time, the unity of "p" with p that is exhibited by the apparently intentional proposition is *not* a self-conscious one, but rather just the unity of disguotation that is already exhibited by exhibiting them together: the proposition and the state of affairs or fact to which it corresponds.³⁴ On the left hand side, within the quotation marks, appears a proposition; on the right-hand side a fact. But the proposition is *itself* a fact: namely, the fact of the appearance of its constituent symbols in the relationship in which they appear. In the simple proposition, the relationship of the names corresponds to the relationship of the simple objects named by them. If the objects are so related, the proposition is true; otherwise it is false. So "p' says p'' is a "correlation of facts by means of a correlation of their objects" in that the names appearing in the proposition on the left-hand side are correlated to the objects named on the right-hand side.

If the correlation which underlies the possibility for p itself to appear – in this way – in intentional statements in general is indeed a general one, then it is a correlation between language and reality *as a whole.* We can understand this global unity better by considering how it relates, within the broader "picture" theory of the *Tractatus*, to the sense and possibility of negation. According to *TLP* 4.0621, the signs 'p' and '~p' correspond to the same reality, but with opposite sense: in particular, if 'p' represents the *existence* (the obtaining) of a particular state of affairs, '~p' represents the *nonexistence* (the nonobtaining) of the same state of affairs (4.1) That a state of affairs does *not* exist means that a certain combination of objects involved in it fails to exist: i.e., that the objects fail to be combined in a certain way (in which they would be combined, were p true). Still, these objects *themselves* must exist (2.026-2.03) and their combination in the way indicated by p must be *possible* (their possibilities must be inscribed in the forms of the simple objects themselves) (2.03-2.034) The requirement that the objects themselves exist is the same as the requirement of the determinacy of sense, which makes it possible that a proposition *in general* picture a state of affairs or a logical combination thereof. The way in which p appears uniformly in "p", "~p", and "p' says p" is that the sentence p shows its sense: that is, in saying

³³ The disquotational form evidently suggests a potential connection or identity (which I shall not develop here) to the disquotational form of the T-schema which Tarski offered in 1933 as a univocal criterion for the success of a semantic truth-definition for a specific language L.

³⁴ This is not to say that quotation or disquotation are not, themselves, problematic; as Davidson often points out (e.g. in the early paper "Theories of Meaning and Learnable Languages", where he points out that the apparent possibility of understanding the meaning of quotations *in general* by disquoting conceals a puzzle about the possible basis of any rule underlying this general possibility.

that things stand in a certain way, it shows how they stand *if* it is true (4.022); and also how things are *not*, if it is false.³⁵

Showing its sense, the unitary p shows a logical form – or rather, the form shows itself in the proposition. But (4.1212) what can be shown, cannot be said. So the "proposition" that would say what is *shown* in 'p' – as opposed to what it says, which is just that p - is, in fact, no proposition at all. It is not only that -- as Kimhi indeed underscores -- it is not a proposition that can be understood as any kind of predicative determination of something – of a substance – *in* the world. "It" is not a proposition at all: *a fortiori*, not one that can be said or grasped from *outside* the world, either.

In this way, the *Tractatus* picture theory, which begins by appearing to invoke the correspondence of language and world as their identity of logical form, overcomes itself from within. Attempting to envision a perspective from which the total coordination of language with reality could be instituted or maintained, the perspective from which it would be possible to survey the "correlation of facts by means of a correlation of their objects" that makes all intentionality possible is itself shown to be impossible: not only incapable of expression, but also incapable of being thought at all. More generally, as logical form is inexpressible, It follows from this that *any (seeming) proposition that asserts (or appears to assert) the unity of thinking and being must itself be nonsensical.* And as the (seeming) propositions that seem to assert this unity are nonsensical, so also are the (seeming) propositions that would deny it.

If we wish nevertheless to articulate such a "proposition" (of identity or difference) within the language of the *Tractatus*, however, we are led in a familiar way to *reject* the unity of thinking and being that is (supposed to be) expressed by the univocity of the law of noncontradiction itself. Pretending, for the moment, that there is such a coherent perspective and attempting to articulate such a claim from this perspective: *either* we will assert limit-contradictions (for example: that "the world is all that is the

³⁵ At 3.14-3.141 Wittgenstein says that a propositional sign is itself a fact, and at 3.1432 he writes that "Instead of "The complex sign 'aRb' says that a stands to be in the relation R', we ought to put, "*That 'a*' stands to *'b'* in a certain relation says *that aRb*." In *Thinking and Being*, pp. 101-106, Kimhi calls this view of the propositional sign "factualism" and distinguishes between two varieties of it. On "c-factualism," the view that has most often been attributed to Wittgenstein, the simple proposition gives a spatial (or perhaps quasi-spatial) model of a state of affairs that can be used to make *either* of two contradictory claims: either that the state of affairs corresponding to the sign obtains, or that it does *not* obtain. On "p-factualism," by contrast, the (simple) propositional sign is usable as a model of a state of affairs *only insofar as (and because)* this use incorporates a positive act of) assertion (which can also be negated): namely, it incorporates the assertion that *things stand thus*.

The basis of Kimhi's preference for p-factualism is his claim that the c-factualist cannot describe, in terms of any difference in spatial (or quasi-spatial) models, the difference between a simple proposition and its negation. But actually, the c-factualist, or at any rate Wittgenstein himself, *does* have these means; and so there is no need, in the broader context of the *Tractatus*, to equip him with an additional theory of acts of assertion in Kimhi's sense. For while it is true, as Kimhi underscores, that the difference between *p* and ~*p* is not any difference between two representational contents, this does not mean it cannot be expressed at all within the c-factualist framework. Rather, trivially, it can be expressed thus (as Wittgenstein indeed suggests): where "p" expresses that *p* (is the case), "~p" expresses that it (i.e. the same thing) is *not* the case. This is not any difference of represented content because it is, rather, a difference of, or articulation of, logical form. And (as Wittgenstein writes at 4.0312, stating what he calls his 'fundamental idea') there can be no *representatives* of differences of this kind: no representatives, that is, of the logic of facts.

case" appears both, itself, to be the case and not something that is the case): in this way we will affirm (with the dialetheist) a thinkably contradictory structure of being in itself.³⁶ Or, denying the possibility of *asserting* contradictions, something will *be* (the case) that *cannot* be expressed (or thought in propositional terms) at all.³⁷ Either way, the attempt to assert, describe or characterize at all the *unity* of thinking and being as involving the (thinkable) unity of the law of noncontradiction ends by destroying itself.

But rather than continuing to engage the pretense, we do better -- with the "resolute" interpretation of the Tractatus and in line with the apparently summative suggestion of 6.54 -- to "kick away the ladder" of our (seeming) attempt to characterize "the" relationship of thinking and being itself. In working through, as the *Tractatus* does, the linguistic form of our propositions, we come to see that the seeming propositions that seemed to describe this global relationship were no propositions at all. This includes the "propositions" stating the law of noncontradiction – in both its ontological and psychological forms. It includes, as well, the propositions that articulate for Aristotle the "privilege" of the philosopher as the authority on this (or these) principles and the other "principles of reasoning." Kicking away the ladder, we come to see that the perspective from which these principles appeared globally able to stand in force in determining the forms of our thinking and ensuring their coordination with the world, is also no perspective at all.³⁸ This – that there is no such perspective at all – comes into view only from within the project of attempting to work out the structure of the total relationship between language and reality and reflecting on the limits of possible sense of the kind announced in the preface of the Tractatus, a project of which Aristotle, of course, had (and could have had) no detailed conception. But at the same time, it is then possible to suspect here that it is just Aristotle's ignorance of the possibility and implications of such a reflexively linguistic project that is responsible for his thought that there can be a "science of being qua being" whose thinkable and knowable principles can go beyond those of "nature" or (in Wittgenstein's terms) all that is the case, as this science – if it has the generality Aristotle accords to it -- is required to do.

Ш

In Plato's *Sophist* at 256e, the Eleatic Stranger puts a problem about negation as the problem of the approach of thought to an unlimitedness that is implied in its very structure:

Stranger: And so, in relation to each of the classes (*ton eidon*), being is many, and not-being is infinite in number (*apeiron de plethei*).

The problem is that, with respect to any given predicate, while its positive determination, or the positive determination of an entity as having that predicate or property, may be seen as delimitative, in that it determines the entity or predicate as to what it is, the determination that an entity does *not* have a property, or that a predicate is *not* thus delimited, rather sends us off into an unlimited indeterminacy. For the negative determination without actually determining in any respect what the entity is or what the predicate ranges over, since there are (for each property or class) infinitely many others that are

³⁶ For a treatment of the *Tractatus* that sees it as committed to limit-contradictions in this sense, see Priest (2002, chapter 12).

³⁷ This is the central claim of (what Conant calls) the "ineffability" variant of non-resolute readings of the *Tractatus* (see Conant (2000))

³⁸ For a vivid account of this movement, see, e.g. Diamond (2000).

incompatible with it. So it is hard to see how any negative judgment can be determinative at all, or how anything can be asserted by one.³⁹

Plato's solution to this problem appears to be suggested by the introduction of the idea, at 257c-e, that otherness itself (or, let us say, the Other itself) is *partitioned* by a judgment that something is not-X: that the description of something as *not beautiful* indicates its belonging to a part of the Other that is exactly that "opposed to" the beautiful, the part we may indeed call the not-beautiful (257d). Thus, in a way at least analogous to the way that knowledge itself, though in itself unified, may be partitioned out according to subject matter, so the Other may be understood as having as many parts as there are negative determinations of something as being not-X, for each possible X.⁴⁰ The partition will be, on the one hand, a definition and articulation of the space of *logically possible judgments* (positive or negative) about these subject matters; and on the other hand, an articulation of the capability of the thinking subject of judgment to, making these judgments, determine the space of these subject matters in general.

In the context of the suggestion earlier in the dialogue (249d-e) that being in general can be understood in terms of capacity (*dunamis*), the solution underwrites what we may see as an original conception of the logical *dunamis* or common structure of capacity shared between the soul and the world. In particular, it suggests an account on which the possible sense of both negative judgments and falsehoods may be understood in terms of the *regulated possibility* of association (or mixing) of logically prominent forms or kinds (including, importantly, the kinds Being and Otherness), a potentially which is mirrored in the capacity of the soul in thinking and judging that issues in predicative belief and speech (261c-263e).⁴¹

Now, I do not wish here to consider whether the solution is successful on the terms that Plato himself sets. I wish only to note the conditions that must be in place for this solution to work: that is, for an account of the capacity of judgment underlying our recognition of the predicative unity of the sentence, which admits of negation, to function as the Visitor's solution requires it to. In particular, the Visitor's solution appears to require, for the partitioning of Otherness in this way, a *standing* metaphysics of existent and unchanging forms, types or categories. Thus, for example, the negative judgment such as *Theatetus is not flying* appears to say, of Theaetetus, that *flying* is *different from* all the attributes that hold of him; or that *flying* is incompatible, within a range, of everything that *does* hold of him. For this to be possible in particular, both the attribute or predicate "flying" and its range of application must already be in place, quite prior to any actual activity of judging or predicating that makes use of it.⁴² And more generally, the total partitioning of Otherness that is seen as required by the possibility of negation

³⁹ This problem is not simply the same as Parmenides' problem about non-being, in the sense of the simple problem of how it is possible to speak or think about "what is not": rather, it is explicitly a problem about the structure of negative judgment, *given* the thought (articulated just before this, at 256d) that it is possible for "*that which is not* to be" (in the sense that *that which is not-X* is so in that it is different from X). Compare, here, Kant's treatment of the judgment "The soul is not mortal" at *CPR* A72/B97-A73/B98. See also McDowell (2009) for the distinction between the problems, as well as an argument that Plato is not deeply concerned with the first one. ⁴⁰ In this paragraph, I follow Kimhi's analysis on pp. 138-152 of *Thinking and Being*.

⁴¹ "Regulated" in the sense that it is essential (252d) that not every one has the unlimited potential to mix with every other one, but rather that only some combinations rather than others are possible: there is thus something like a constitutive "logical grammar" of their possible combinations.

⁴² See Davidson (2005), pp. 80-81 for this point.

and which also delimits the total possibilities of knowledge must already be in place as well, prior to any such activity.

It is thus required, for the form of solution that Plato offers to be possible, that there *already be* in place a total metaphysics of standing forms and their *determinate logical relations* that essentially precedes the existence and life of actual speakers. In this respect, the picture Plato suggests closely resembles the metaphysics of the *Tractatus* if read "straight" – for as we saw in the last section, that metaphysics itself requires a total partitioning of logical space by means of the possible interrelations of standing and sempiternal objects (in the *Tractatus*, the simple objects themselves) and their intrinsic logical possibilities of combination.

But does the general form of explanation that they attempt to provide still make sense, outside the context of this assumption? That is, can we still suppose that thinking and being can be unified by a general form of the analysis of the capacities involved in thinking (in the sense of judging) if we *do not* presuppose such a metaphysics, but rather situate our understanding within the actual lives of embodied, finite learners and speakers of a natural language?

To see why it does not, it is helpful first to note the significant *non-triviality* of the semantic knowledge that such a speaker comes to possess in learning and coming to speak their first language. In the case of actual speakers this learning is not a matter of simply acquiring an *a priori* framework of logical relations that are prepared in advance. Rather, it is marked by (among other things) coming to appreciate and integrate a vast amount of "a posteriori" knowledge about the reference of names, the range of predicates, and more broadly the use of expressions across a vast and practically undelimitable *diversity* of occasions and situations .

Now, as we have seen, the mark of a *general* capacity for judgment, in the sense required to vindicate the sameness of thinking and being, is plausibly the capacity to recognize the "contradictory unity" of *p* and ~*p*: for example, to recognize something of the form "*p* and ~*p*" as empty – and thus incapable of being judged -- for *any p* formulated in terms of concepts the speaker understands. Of course, a speaker who is "competent" in the use a language including an indicator of negation such as "not" will recognize "p and not p", stated that way, as a contradiction. But as a matter of actual use, contradictions in general will by no means generally "come" marked in this way. For example, in "Theaetetus sits and Theaetetus flies;" or "The morning star rises and the evening star does not rise" we may *eventually* come to be in a position to detect the contradiction. But to do so will require further and essentially *a posteriori* investigations, both of the world and of our own usage, in order that we come into that position. And at any rate, that position is in *no* plausible sense attributable to a competent speaker simply on the basis of their competence in the use of the component concepts.⁴³

Whatever the prospects, then, for eventually coming to recognize such "concealed" contradictions as such, it is evidently impossible to arrive at this recognition by means of an *a priori method or structure of logical analysis alone.* As he makes clear in the 1929 (transitional) article "Some Remarks on Logical Form," a recognition of the impossibility of establishing the unity of propositions without a partially *a posteriori* analysis of the structure of phenomena was one of the major initial motivations of

⁴³ If there indeed *is* one: in the first case, as Davidson points out in a different context, there is an evident sense in which the sentence has been rendered non-contradictory by the advent of modern air travel.

Wittgenstein's abandonment of the *Tractatus* thesis of the independence of atomic propositions, and (with it) of the method of analysis it proposes:

Now we may only substitute a clear symbolism for the unprecise one [of ordinary language – P.L.] by inspecting the phenomena which we want to describe, thus trying to understand their logical multiplicity. That is to say, we can only arrive at a correct analysis by, what might be called, the logical investigation of the phenomena themselves, *i.e.* in a certain sense *a posteriori*, and not by conjecturing about *a priori* possibilities. ⁴⁴

With this, we are apparently at least on the way to the *Philosophical Investigations'* skepticism about the possibility of a "unique" analysis and its different methodology of reflection on the multiplicity of language games. But if anything we could present as the basis of an understanding of the unity and functioning of propositions (in judgment, and elsewhere) requires a *posteriori* investigation in this sense, its possibility *in general* cannot be attributed to the capacities of any specific rational subject, or indeed to rational subjects in general *qua* their possession of such capacities themselves.

Within the context of an attempt to defend the generality of judgment as grounded in the capacity of subjects, it might be objected that the attainment of semantic knowledge in this sense is not a matter of the exercise of (the capacity of) judgment itself, but is only -- at most -- a matter of "concept acquisition" in the sense that the relevant capacity can operate with generality on the concepts (here, the concepts "sitting" and "flying") *once* those concepts are "completely" acquired. But we have then a right to ask the proponent of such a view what it could mean for a concept to be "completely" acquired in this sense. And we should note, then, both that: i) it is not plausibly a part of the ability of speakers competently to use *any* concept that they *can* use, that they are able to determine, for *each* use of it, whether or not its use in that context and reference excludes, or does not exclude, its use in *any* and *all* others; and ii) there is plausibly *no* concept for which the acquisition of (at most) finitely much semantic information in this sense.⁴⁵

In relation to the last point in particular, additionally, there is a compelling formal reason to think that *no* ability of a finite learner and speaker of language *could* – even in principle – be a general capacity of judgment of the form we have been discussing, *even if* that speaker is seen as possessing *any* amount of semantic information of this kind. For (recall once more) a general capacity of judgment in this sense would also be an ability to discern the "contradictory unity" of *p* and ~*p* for *arbitrary* propositions *p* (that the speaker understands) – to discern, that is, a contradiction, *whenever* it occurs. On standard assumptions of metalogic and formal semantics, such a sentence is "unsatisfiable" in that cannot be true, on any assignment of referential values to its nominative terms. As the negation of a sentence that is unsatisfiable in this sense is a logical "validity" or tautology, someone equipped with an ability to detect whether a sentence is satisfiable would also possess an ability to detect, in general, whether a

⁴⁴ Wittgenstein (1993), p. 30.

⁴⁵ To insist upon this is to insist, of course, in other terms on the problem of the rule and its application that is one centrepiece of Wittgenstein's argument against the idea that "if anyone utters a sentence and *means* and *understands* it, he is thereby operating a calculus according to definite rules" in the *Philosophical Investigations (PI* 82). For to suppose otherwise would be to suppose that there could be a (finitely stateable) rule that determines all its instances (and what is not an instance) by itself.

sentence is a validity in this sense.⁴⁶ But by the near-simultaneous results of Church and Turing published in 1936 and 1937, *there can be no decision procedure* for determining logical consequence, and hence no decision procedure for discerning (first-order or higher) logical validities in general. Accordingly, there can *also* be no decision procedure for recognizing the satisfiability or unsatisfiability (i.e. contradictoriness) of arbitrary sentences, *even* given information about the relevant topics that is as "complete" as one likes.⁴⁷

As there cannot be a decision procedure for determining satisfiability, and there cannot be a decision procedure for logical consequence, there *cannot be a capacity for judgment* that is general in the sense we have been considering. The result, put this way, admittedly requires one further assumption: namely, that someone who possessed (*per impossible*, as I am arguing) a rational capacity to perform a task would *also* (thereby) possess an "effective" decision procedure for performing that task. But that assumption is sufficiently motivated by the idea of an effective procedure itself. This idea is, in essence, the idea of a procedure that could be followed by a finite subject of language, capable of learning a natural language in a finite amount of time and possessing only finitely much information, to arrive at a decision, on the basis of principles which *themselves* can be cited, articulated, and recognized as valid.⁴⁸ But this idea is plausibly just the idea of what makes any procedure a "rational" one at all. If, by contrast, there is a procedure that is not articulable in terms of principles in this sense, it is not clearly a rational one at all.

It follows, then, that there is no capacity attributable to finite speakers that can be understood as a capacity for judgment *in general*. If "thinking" means the rational activity of finite speakers of language, there is no defensible conception of this thinking that would validate the claim of its formal identity with the totality of all that is, or can be, the case.

Could "thinking" relevantly mean something else, however? For example, granting this, could our own capacities not nevertheless be seen as *limited* varieties of an *unlimited* capacity attributable to infinite knowers (for example to *nous* or the Absolute Spirit), thinkers not subject to the relevant constraints of our finitude and thereby able to possess the relevant kind of capacity in its full and general sense?

The answer is that they *cannot* be so seen. For if there were such a thing as a knower whose activity in coming to know is unlimited in this sense, the "capacity" (if we can call it such) it deploys in this activity would, again, be no *rational* one. It would not be, that is to say, a capacity whose exercise stands under the constraint of the need to justify itself by citing rational principles by which it goes: indeed, conceived as a power for knowledge that is unlimited in the requisite sense, it would stand under no need to go by principles at all. At best, it could be – as Kant says in the first *Critique* -- the intellectual intuition of an intellect co-original with the world itself, one capable of intuiting the unity of its knowledge with the world without standing under any need whatsoever to discern, discover, or produce this unity.⁴⁹ For

⁴⁶ Church (1936) and Turing (1937)

⁴⁷ I am indebted to Jon Cogburn for (part of) the suggestion to put the formal issues in these terms.

⁴⁸ In the context of truth-theories in the style of Davidson, this is also the constraint that yields the requirement that (any reasonable) truth-theory that can be seen as embodying a speaker's understanding of the meaning of a natural language must be recursively enumerable (see, e.g., Davidson (2001)). Alternatively, the relevant additional assumption might be seen as a version of the "Church-Turing thesis" about effective procedures and solvable problems.

⁴⁹ Kant, *CPR*, B145-146.

such a "subject," the logic that articulates the possible forms of judgment in their relationship to the world has no significance, and its powers of cognition are no powers of judgment – no powers of knowing facts of the form a is ϕ – at all.

If "thinking" does not mean the bare intuition of such an absolute intellect, then, thinking (in any reasonable sense of the term) and being are not the same. Seeing this, it becomes possible to see the claim of sameness more as the *presupposition* of a certain kind of (historically prominent) logical-metaphysical project than any actual result of that project or any motivated by it. But at the same time, the analysis of the actual possibility of logical systems as they are plausibly available to real speakers whose competence can be captured in finite terms evinces a "deeper" and more characteristic *undecidability* of (anything we may understand as) "the" "thinking-being relationship" itself. Appreciating this undecidability, it is no longer possible to envision the human animal as defined by its possession of a capacity for rational judgment or by its self-conscious possession of the *logos* in general. But at the same time, it becomes possible to inquire critically into the actual constitution of the idea of consistent unity that has been the very institution of one version of this animal's claim to dominance in general over all the others of the earth.

Faced with the demand that a form of expression appears to place on us, we may answer that demand on the level of its own apparent force, in a way correlative (so to speak) to the fascination that the form exerts over us. Or, alternatively, we may ask after the conditions of the appearance of that seeming demand, its motivation, and its constitution -- thereby also asking what other claims (of ourselves on ourselves, or on others) it covertly promotes or protects. In this case, the investigation becomes a selfinvestigation. In it, we may come to find that what appeared as our need for solution was not at all what it seemed: that the form of unity that seemed to be demanded was one that led us to mistake our real need, and in so doing to mistake ourselves. So finding, we clear away the ground, and allow ourselves to guess that the form we sought may lie somewhere completely else, in a quite different connection to our own lives, and to our lives with others. The constitution of the idea of solution, along with the problem itself, seemed to serve a need of unity; but it is now open to us to ask whether *that* need was ever really ours at all.

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