## Paul M. Livingston Realism and the Infinite<sup>1</sup>

"Not empiricism and yet realism in philosophy, that is the hardest thing." —Wittgenstein

"A human is that being which prefers to represent itself within finitude, whose sign is death, rather than knowing itself to be entirely traversed and encircled by the omnipresence of infinity." —Badiou

## I

In his 1951 Gibbs lecture, drawing out some of the "philosophical consequences" of his two incompleteness theorems and related results, Kurt Gödel outlines a disjunctive alternative which, as I shall try to show, captures in a precise way the contemporary situation of reflective thought in its ongoing consideration of the relationship of formalism to the real:

Either mathematics is incompletable in [the] sense that its evident axioms can never be comprised in a finite rule, i.e. to say the human mind (even within the realm of pure mathematics) infinitely surpasses the powers of any finite machine, or else there exist absolutely unsolvable Diophantine problems of the type specified...<sup>2</sup>

A consequence of this aporeatic situation of contemporary thought, as I shall try to show, is that the longstanding philosophical debate over the relative priority of thought and being that finds expression in discussions of "realism" and "anti-realism" (whether of idealist, positivist, or conventionalist forms) can only be assayed from the position of a metaformal reflection on the relationship of the forms of thought to the real of being. Moreover, if Gödel's argument is correct and can be generalized beyond the epistemology of mathematics itself, it is also not neutral on this question of relative priority, but rather suggests a new kind of realism—what I shall call "metaformal" realism—that differs markedly both from "metaphysical realism" and from the newer varieties of "speculative realism" on offer today.

The type of realism I shall defend here is not primarily a realism about any particular class or type of objects or entities. Thus it is not, a fortiori, an empirical realism or a naturalism (although I also do not think it is inconsistent with positions that march under these banners).<sup>3</sup> Its primary source is not any empirical experience but rather the experience of formalization, both insofar as this experience points to the real-impossible point of the actual *relation* of thinkable forms to being and insofar as it schematizes, in results such as Gödel's, the intrinsic capacity of formalization problematically to capture and decompose its own limits. In The Politics of Logic, I systematically interrogated the consequences of formalism and formalization in this sense for contemporary political, social, and intersubjective life according to the various orientations possible today for thought in its total relation to being, seeking to locate, in each case, the actual point and limits of the effective formal capture of the real in thought. In particular, I suggested there that both of the orientations I presented as "post-Cantorian" demand a realist attitude grounded in this experience of the transit of forms, and capable of acknowledging their inherent difference from anything simply created or *produced* by finite human thought. Accordingly, I believe the metaformal realism I shall develop more fully here might be formulated precisely, referring in passing to the Lacanian motto according to which "the Real is the impasse of formalization," as a realism of the "Real" in something like Lacan's sense—that is, in the sense in which it represents both an inherent limit-point and an obscurely constitutive underside for both of the other two "registers" of the Imaginary and the Symbolic.4

To arrive at the disjunctive conclusion he draws in the lecture, Gödel draws on a concept central to twentieth-century inquiry into the foundations of <u>mathematics</u>, that of a "finite procedure." Such a <sup>3</sup> I return to the issue of the relationship of realism to materialism in section IV, below.

<sup>&</sup>lt;sup>1</sup> I would like to thank Reuben Hersh and John Bova for discussions of the issues in this paper. A longer and more comprehensive version is available at: http://www.unm. edu/-pmliving/

<sup>&</sup>lt;sup>2</sup> Kurt Gödel, "Some basic theorems on the foundations of mathematics and their philosophical implications" in *Unpublished Philosophical Essays*, ed. Francisco A. Rodriguez-Consiguera (Basel/Boston/Berlin: Birkhäuser Verlag, 1995), 134.

<sup>&</sup>lt;sup>4</sup> Of course, Lacan's concept of the "Real" is complex and undergoes many changes of specification and inflection over the course of his career. I do not take a view here about how precisely to define it or which formulation is most important, but seek only to preserve the link that is constitutive for Lacan between the Real and formalization at the latter's point of inherent impasse. For a very exhaustive and illuminating treatment of Lacan's concept, see Tom Eyers, *Lacan and the Concept of the "Real"* (London: Palgrave MacMillan, 2012). I also discuss Lacan's motto and Badiou's reversal of it into his own claim for a "theory of the pass of the real, in the breach opened up by formalization..." in Paul Livingston, *The Politics of Logic: Badiou, Wittgenstein, and the Consequences of Formalism* (New York: Routledge, 2012), 188-192.

procedure is one that can be carried out in a finite number of steps by a system governed by well-defined and finitely stateable rules, a so-called "formal system." The significance of the investigation of formal systems for research into the structure of mathematical cognition and reality lies in the possibility it presents of rigorously posing general questions about the capacities of such systems to solve mathematical problems or prove mathematical truths. For instance, one can pose as rigorous questions i) the question whether such a system is capable of proving all arithmetic truths about whole numbers; and ii) whether such a system is capable of proving a statement of its own consistency. Notoriously, Gödel's first and second incompleteness theorems, respectively, answer these two questions, for any consistent formal system capable of expressing the truths of arithmetic, in the negative: given any such system, it is possible to formulate an arithmetic sentence which can (intuitively) be seen to be true but cannot be proven by the system, and it is impossible for the system to prove a statement of its own consistency (unless it is in fact inconsistent).

Gödel's argument from these results to his "disjunctive conclusion" in the lecture is relatively straightforward. The first incompleteness theorem shows that, for any formal system of the specified sort, it is possible to generate a particular sentence which we can "see" to be true (on the assumption of the system's consistency) but which the system itself cannot prove.<sup>5</sup> Mathematics is thus, from the perspective of any specific formal system, "inexhaustible" in the sense that no such formal system will ever capture all the actual mathematical truths. Of course, given any such system and its unprovable truth, it is possible to specify a *new* system in which that truth is provable; but then the new system will have its own unprovable Gödel sentence, and so on. The question now arises whether or not there is some formal system which can prove all the statements that we can successively see to be true in this intuitional way. If *not*, then human mathematical cognition, in perceiving the truth of the successive Gödel sentences, essentially exceeds the capacities of all formal systems, and mechanism (the claim that human mathematical coginition is, or is capturable by, a formal system) is false; this is the first alternative of Gödel's disjunction. If so, however, then there is some formal system that captures the capacities of human mathematical thought. It remains, however, that there will be statements that are undecidable for this system, including the statement of *its* consistency, which is itself simply an arithmetical statement. In this case there are thus problems that cannot be solved by any formal system we can show to be consistent *or* by any application of our powers of mathematical cognition themselves; there are well-defined problems that will remain unsolvable, now and for all time.<sup>6</sup>

The two options left open by Gödel's disjunctive conclusion correspond directly to the two post-Cantorian orientations of thought, or positions on the relation between thought and being, that I called in The Politics of Logic the "generic" and "paradoxico-critical" orientations.7 On the first of Gödel's disjunctive options, the power of the human mind to grasp or otherwise comprehend truths beyond the power of any finite system effectively to demonstrate witnesses an essential incompleteness of any finitely determined cognition and a correlative capacity on the part of human thought, rigorously following out the consequences of the mandate of consistency, to traverse by means of a "generic" procedure the infinite consequences of truths essentially beyond the reach of any such finite determination. On the second of the options, the essential indeterminacy of any such system witnesses, rather, the necessary indemonstrability of the consistency of any procedural means available to the human subject in its pursuit of truth, and thereby to the necessary existence of mathematical problems that are absolutely unsolvable by any specifiable epistemic powers of this subject, no matter how great. Both orientations, as I argued in the book, as well as the necessity of the (possibly non-exclusive) decision between them, result directly from working through the consequences of the systematic availability of the infinite to mathematical thought, as accomplished most directly through Cantor's set theory and its conception of the hierarchy of transfinite cardinals. More broadly, as I argued in the book, what is most decisive for the question of the orientations available to thought today is the consequences of the interlinked sequence of metamathematical

<sup>&</sup>lt;sup>5</sup> I here state the first theorem, roughly and intuitively, appealing to a notion of "truth" that is in some ways problematic. For discussion of the issues involved in the difference between this and other, less potentially problematic statements, see Livingston, *The Politics of Logic*, chapter 6.

<sup>&</sup>lt;sup>6</sup> The result that Gödel refers to in 1951 is that the consistency statement for each particular system is equivalent to some statement of the form:  $\forall x_1...x_n \exists y_1...y_m [p(x_1,...,x_n, y_1,...,y_m) = o]$  where *p* is a polynomial with integer coefficients and the variables range over natural numbers; later the work of Davis, Putnam, Robinson and Matiyasevich showed that one can replace the statement with something of the form:  $\forall x_1,...x_n [p(x_1,...,x_n) \neq o]$ 

<sup>&</sup>lt;sup>7</sup> For the four orientations, see Livingston, The Politics of Logic, 51-60.

and metalogical reflection running from Cantor, through Gödel's incompleteness theorems, up to Cohen's demonstration of the independence of the Continuum Hypothesis from the axioms of ZF set theory; it is thus not surprising that Gödel's own "philosophical remarks" about the implications of his own results should replicate the general disjunction in a clear and specific form.

Gödel himself, in the lecture and elsewhere, was concerned to draw out the implications of his own result for the hypothesis of mechanism; as subsequent discussion has made clear, though, it is in fact problematic for many reasons, including the unclarity of the mechanist thesis itself, to argue directly against (or for) mechanism simply on the basis of metamathematical results.8 Notwithstanding this, it is possible to see the upshot of Gödel's "disjunctive conclusion" in the lecture as bearing relevance, beyond the issue of mechanism as well as the confines of "philosophy of mathematics" narrowly construed, to somewhat different philosophical issues.<sup>9</sup> In particular, it points to a distinctive and non-standard, but comprehensive position of *realism*, what I shall call *metaformal* realism.<sup>10</sup> For this realism, the decisive issue is not, primarily, that of the reality of "mathematical objects" or the possibility of understanding them as determinate independently of the routes of access to them (epistemic or otherwise) involved in the exercise of our human capacities. It is, rather, that both terms of Gödel's disjunction capture, in different ways, the

<sup>9</sup> I thus follow Feferman in considering that, even if there are problems with applying Gödel's reasoning directly to the question of mechanism, "...at an informal, non-mathematical, more every-day level, there is nevertheless something to the ideas involved [in his argument for the "disjunctive conclusion"] and something to the argument that we can and should take seriously." Solomon Feferman, "Are there absolutely unsolvable problems? Gödel's dichotomy." Philosophia Mathematica 14:2: 11 (page # reference to on-line version at: http://math.stanford.edu/-feferman/ papers/dichotomy.pdf).

<sup>10</sup> In *The Politics of Logic*, I called this position simply "formal realism." I add the prefix "meta-," here, to reflect that what is concerned is not primarily an attitude (e.g. a Platonist one) about the "reality" or "actual existence" of forms, but rather the implications of the *transit* of forms in relation to what is thinkable of the real, the transit that can, in view of Cantor's framework, be carried out beyond the finite. structural point of contact *between* these capacities and what must, on *either* horn of the distinction, be understood as an infinite *thinkable* structure determined quite independently of anything that is, in itself, finite. Thus, each term of Gödel's disjunction reflects the necessity, given Gödel's theorems, that any specification of our relevant capacities involve their relation to a structural infinity about which we must be realist, i.e. which it is not possible to see as a mere production or creation of these capacities.

On the first alternative, this is obvious. If human mathematical thought can know the truth of statements about numbers which are beyond the capacity of *any* formal system to prove, then the epistemic objects of this knowledge are "realities" (i.e. truths) that also exceed any finitely determinable capacity of knowledge. But on the second alternative, it is equally so. If there are well-specified mathematical problems that are not solvable by any means whatsoever, neither by any specifiable formal system nor by human cognition itself, then the reality of these problems must be thought of as a fact determined quite independently of our capacities to know it (or, indeed, to solve them).<sup>11</sup> On this alternative, we must thus acknowledge the existence of a reality of forever irremediable problems whose very issue is the inherent undecidability that results from the impossibility of founding thought by means of an internal assurance of its consistency. In this way the implications of the mathematical availability of the infinite, on either horn of the disjunction, decompose the exhaustiveness of the situation underlying the question of realism and idealism in its usual sense: that is, the question of the relationship of a presumptively finite thought to its presumptively finite object.

The metaformal realism thus indicated has several further distinctive features, which I briefly adumbrate:

1. Metaformal realism is not a "metaphysical realism." In particular, because it is grounded solely in an *internal* experience of the progress of forms to the infinite, it avoids any need to posit an empirical or transcendent referent beyond the effectiveness of

<sup>&</sup>lt;sup>8</sup> Thus, for instance, in a recent very comprehensive review of discussion about Gödel and mechanism, Stuart Shapiro concludes that "there is no plausible mechanist thesis on offer that is sufficiently precise to be undermined by the incompleteness theorems." Stuart Shapiro, "Incompleteness, mechanism, and optimism," *The Bulletin of Symbolic Logic* 4:3, September 1998: 275.

<sup>&</sup>lt;sup>11</sup> Gödel says this about the second term of the disjunction: "... the second alternative, where there exist absolutely undecidable mathematical propositions, seems to disprove the view, that mathematics (in any sense) is only our own creation...So this alternative seems to imply that mathematical objects and facts or at least *something* in them exist objectively and independently of our mental acts and decisions, i.e. to say some form or other of Platonism or "Realism" as to the mathematical objects." *Kurt Gödel: Unpublished Philosophical Essays*, 135-136.

forms and formalization and does not ground its realism in any such referent. It is thus completely distinct from any realism of a "mind-independence" variety, which always requires a problematic doctrine of the bounding of thought in relation to its empirical objects. It also does not require, and does not encourage, the possibility of a "view from nowhere" or a "single unique description of reality."

2. Metaformal realism is a *reflective*, not a "speculative," idealism. It develops all of its consequences internally, from internal reflection on the limitology of thought and its inherent formal features. It thus has no need to posit an object of *speculation* simply external to this limitology or to engage in the uncertain investigation of the features of such an object. If it is, as I shall try to show, engaged in an inherent dialectic of thought with being, this dialectic is thus not a *speculative* dialectic of "determinate negation."<sup>12</sup>

3. Metaformal realism de-absolutizes the world as a transcendent object of thought. As I argued in The Politics of Logic, the twentieth-century inquiry into forms has the consequence of consigning formal thought about the totality of the world (indeed, thought about totality in general) to an unavoidable disjunction, what I called there the "metalogical duality" between consistent incompleteness and inconsistent completeness, essentially the same alternatives involved in Gödel's "disjunctive" conclusion. This means, as well, a basic diremption of any figure of thought that countenances a (complete and consistent) Absolute, and forces a choice between acknowledging the essential incompleteness of consistent thought or countenancing the existence of the totality of the world only under the heading of the reality of the inconsistent.

Π

In contemporary philosophical discourse, no project has done more to illuminate the issue of realism and its underlying formal determinants than Michael

Dummett's. Familiarly, in a series of articles and books beginning in 1963 with the article "Realism," Dummett has suggested that the dispute between realism and anti-realism with respect to a particular class of statements may be put as a dispute about whether or not to accept the principle of *bivalence* (i.e., the principle that each statement is determinately true or false) for statements in the class concerned.<sup>13</sup> Though this issue yields differing consequences in each domain considered, the acceptance of bivalence generally means the acceptance of the view that all statements in the relevant class have truth values determined in a way in principle independent of the means and methods used to verify them (or to recognize that their truth-conditions actually obtain when they, in fact, do so); the anti-realist, by contrast, generally rejects this view with respect to the relevant class. Dummett did not envisage that this comprehensive framework would or should support a single, *global* position of metaphysical "realism" or "anti-realism" with respect to all domains or the totality of the world; rather, his aim was to illuminate the different kinds of issues emerging from the traditional disputes of "realism" and "idealism" in differing domains by submitting them to a common, formal framework.<sup>14</sup> From the current perspective, however, it is just this aspect of formal illumination which is the most salutary feature of Dummett's approach. For by formally determining the issue of realism with respect to a given domain as one turning on the acceptance or nonacceptance of the (meta-)formal principle of bivalence with respect to statements, Dummett points toward a way of conceiving the issue that is, in principle, quite independent of any ontological conception of the "reality" or "ideality" of objects of the relevant sort.

Although this kind of consideration finds application quite generally, it is certainly no accident that the historical dispute which forms the basic model for Dummett's formal framework itself is the dispute between formalists and intuitionists about the foundations of mathematics in the 1920s and early 1930s. Partisans of the two positions reached deeply opposed conclusions about the nature of reasoning about the infinite, but for both positions the idea of a *finite (i.e., finitely specifiable) procedure* or process of demonstration plays a central role. In particular, whereas the formalist position allows

<sup>&</sup>lt;sup>12</sup> I refer here, in passing, to the distinction between "reflection" and "speculation" drawn by Hegel in the "Preface" to the *Phenomenology of Spirit*, **9**59. That I thus distinguish the post-Cantorian orientations of metaformal realism from Hegel's pre-Cantorian speculative dialectic should not exclude that metaformal realism, particularly in its paradoxico-critical variant, nevertheless exhibits a number of important parallels to aspects of Hegel's system, particularly in its treatment of the nature of contradiction *prior* to its dialectical sublation or resolution; for discussion of some of these relationships to Hegel, see *The Politics of Logic*, 253-254.

<sup>&</sup>lt;sup>13</sup> Michael Dummett "Realism" in *Truth and Other Enigmas* (London: Duckworth, 1978), 145-165; for some later reflections on the development of the framework and issues related to it, see Dummett's preface to *Truth and Other Enigmas*.

<sup>&</sup>lt;sup>14</sup> Dummett, Truth and Other Enigmas, xxx-xxxii.

the axioms and rules of a formal system to be extended classically, by means of such a procedure, to arbitrarily extended reasoning about the infinite *provided* that the system can be shown to be consistent, intutionism generally restricts the positive results of mathematics about the infinite to what can be shown by means of a finite, constructivist procedure of proof.

In the 1973 article "The Philosophical Basis of Intuitionistic Logic," Dummett considers the question of what rationale might reasonably serve as a basis for replacing classical logic with intuitionistic logic in mathematical reasoning (hence, in his framework, for replacing realism with anti-realism).<sup>15</sup> As Dummett emphasizes here, the decision between realism and anti-realism depends ultimately on our conception of how sense is provided for mathematical statements, and in particular whether we can conceive of these statements as having sense quite independently of our means of recognizing a verification of them. It is thus, ultimately, general issues about the capacities or practices that we learn in learning a language and deploy in speaking one that determine, given his framework, equally general issues about whether realism or anti-realism is better justified in any given domain. As in the earlier article "Realism," Dummett here emphasizes that this primary issue is not an epistemic or ontological, but rather a semantic one. Thus, "Any justification for adopting one logic rather than another as the logic for mathematics must turn on questions of meaning"; and again, "it would be impossible to construe such a justification [i.e. for adopting classical or intuitionistic logic] which took meaning for granted, and represented the question as turning on knowledge or certainty."<sup>16</sup>

By posing the issue of realism vs. anti-realism, not only in the mathematical case but more generally, as turning on the question of the provision of sense, Dummett shows that the question of realism in a particular domain is most intimately related, not to the question of the ontological status of, or our epistemological access to, its objects, but rather to the question of the coherence and range of the procedures by means of which the meanings of statements about the domain are learned and manifested. But this is none other than, again, the question of the way that the infinite becomes available on the basis of a finite procedure. And it is just here, with regard to the specific question of what is involved in the learning and pursuit of a finite procedure, that the possibility of metaformal

reflection of the sort that I have portrayed Gödel as engaging in proves to be decisive. For Gödel's own incompleteness theorems, of course, result directly from a rigorous metaformal consideration of the range and capacities of formal systems (in Hilbert's sense and related ones). In particular, Gödel's first theorem shows that for any such system, there will be a number-theoretical sentence that is beyond its capacity to prove or refute, and the second theorem shows that no such system can prove its own consistency (assuming that it is consistent). In this way Gödel's results render the formalist conception of finite procedures unsuitable for anyone who wishes to assert solely on its basis the realist position that all the statements of number theory have determinate truth-values, independently of our ways of verifying them; but on the other hand, in invoking under the heading of the "inexhaustibility" of mathematics an essential reference to a reality that marks the point of impasse of any given finite procedure, Gödel's argument shows the intuitionist strictures to be untenable as well.

Just as Gödel's theorems themselves thus overcome the debate between intuitionism and formalism, narrowly construed, by conceptually fixing and reflecting upon the contours of a central concept (that of a finite procedure) commonly appealed to by both, the metaformal realism I have discussed as suggested by Gödel's argument provides a new basis for critically interrogating the central concept of a *rule* of use, as it figures in both "realist" and "anti-realist" conceptions of the structure of language. As I argued in more detail in The Politics of Logic, in particular, it is then apparently possible to draw, with respect to our actual practices and institutions of linguistic use, a conclusion directly analogous to that drawn by Gödel with respect to mathematical reasoning specifically: namely that *either* the consistency of our regular practices can only be known, and assured, by a deliverance of an essentially irregular insight that essentially cannot be subsumed within them or determined by them insofar as they can be captured by rules; or it cannot be known at all and thus can only be treated as a perpetually deferred problem. On either assumption, the claim of consistency is shown to be, from the perspective of the regular provision of sense, the point of an impossible-Real that always escapes, drawing along with it any possibility of an internal systematic confirmation of the infinite noncontradictory extensibility of the rule to ever-new cases. It is in this way, as I have argued, that the phenomenon that Gödel calls the "inexhaustibility of mathematics" points toward a metaformally justified realism of the impossible-Real, correlative to what we may describe as our essential

<sup>&</sup>lt;sup>15</sup> Dummett, "The philosophical basis of intuitionistic logic" in *Truth and Other Enigmas*, 215-247.

<sup>&</sup>lt;sup>16</sup> Ibid., 215.

openness toward the infinite and based in metaformal reflection about the limits and transit of forms. In so doing, it unhinges any possible claim of the humanistically conceived "finite" subject finally to ground itself, or to secure by its own means the ultimate sense of its language and life.

III

For the thinkers and positions that have characterized themselves, over the last few years, as "speculative realist", the work of Quentin Meillassoux has been seen as an inspiration.<sup>17</sup> Much of the influence of Meillassoux's work has derived from the force of his critique, in After Finitude, of what he calls "correlationism."18 Correlationism is, according to Meillassoux, the position that holds that "we only ever have access to the correlation between thinking and being, and never to either term considered apart from the other" and furthermore that the "correlation so defined" is "unsurpassable".19 Meillassoux does not specify the kind of "correlation" figuring in this position as any one type of relation; but he suggests that "the subject-object correlation," "the noetico-noematic correlation," and the "language-referent correlation" may all be treated as examples of the kind of relation with which he is critically concerned.<sup>20</sup> To all of the varieties of correlationism, Meillassoux raises a single objection, that of what he calls the "ancestral." Correlationism in any of its forms, he suggests, cannot account for the existence of a "reality anterior to the emergence of the human species." According to Meillassoux, the correlationist cannot account for an anterior reality in this sense because, in considering it, he must insist upon a "retrojection" of the past on the basis of the present" whereby "it is necessary to proceed from the present to the past, following a logical order, rather than from the past to the present, following a chronological order;" this requires him to hold that "it is not ancestrality which precedes givenness, but that which is given in the present which retrojects a seemingly ancestral past."<sup>21</sup> To this apparent doubling of meaning in

the correlationist's treatment of the arche-fossil, Meillassoux opposes the maxim of what he calls an "irremediable realism": that an ancestral statement "either …has a realist sense, and *only* a realist sense, or it has no sense at all."<sup>22</sup>

Meillassoux's argument against correlationism has been aptly criticized elsewhere for the apparently "straw" character of the figure of the "correlationist" which it invokes; for example, as Peter Hallward points out, even as characteristic an idealist as Husserl in fact only considers claims about the "correlation" of thought or consciousness and objects within an attitude of bracketing claims about their existence in order to consider their sense (rather than, for instance, attempting to explain or derive their existence).<sup>23</sup> This and similar considerations about what is involved in actual idealist positions, including those of Kant himself, may lead us to conclude, with Hallward, that Meillassoux has, in constructing his critique of correlationism, essentially committed an equivocation of epistemological considerations with ontological or semantical ones. On the other hand, Meillassoux at least sometimes suggests that what is decisive for the correlationist position as he is portraying it is an order of precedence that is primarily *neither* epistemological nor ontological, but rather logical or semantic: thus, for instance, in describing the temporal "retrojection" that the correlationist must perform, he describes it as substituting a "logical" for a "chronological" order, and at least at one point he specifies the problem which the correlationist must answer as the problem of the possibility of the meaningfulness of scientific statements about the past.<sup>24</sup> If we take this last suggestion seriously, it might be possible to see the main concern of Meillassoux's argument as turning not on the ontological issue of the existence of objects, or the epistemological one of the conditions for our knowledge of them, but rather on the question of the basis of the provision of sense for sentences about the ancestral past. In this way, Meillassoux could be construed as avoiding the equivocation between epistemology and ontology of which Hallward accuses him; and if construed this way, Meillassoux's argument would approach more closely both Dummett's framework for discus-

<sup>&</sup>lt;sup>17</sup> See, e.g., Levi Bryant, Nick Srnicek, and Graham Harman (ed.), *The Speculative Turn: Continental Realism and Materialism* (Melbourne: re.press, 2011), 3-4. Meillassoux prefers the label "speculative materialism" for his own work.

<sup>&</sup>lt;sup>18</sup> Quentin Meillassoux, After Finitude: An Essay on the Necessity of Contingency, trans. Ray Brassier (London: Continuum, 2006), 5. In the longer version of this paper, I discuss, as well, Meillassoux's more recent article, "Iteration, reiteration, repetition: A speculative analysis of the meaningless sign."

<sup>&</sup>lt;sup>19</sup> Ibid., 5.

<sup>20</sup> Ibid., 6.

<sup>&</sup>lt;sup>21</sup> Ibid., 16.

<sup>&</sup>lt;sup>22</sup> Ibid., 17.

<sup>&</sup>lt;sup>23</sup> Peter Hallward, "Anything is possible: a reading of Quentin Meillassoux's After Finitude," in The Speculative Turn, 137.
<sup>24</sup> Meillassoux, After Finitude, 9. In his critical response to Hallward's critique of Meillassoux, Nathan Brown makes this a central point of contention, charging that Hallward "...ignores Meillassoux's critique of logical retrojection altogether." Nathan Brown, "The speculative and the specific: On Hallward and Meillassoux," in The Speculative Turn, 143.

sion of realism and anti-realism and the position of metaformal realism I am recommending here.

But even following this suggestion, it is not at all evident how to interpret Meillassoux's "correlationist" as an anti-realist in anything like Dummett's sense. For example, though Dummett has discussed within his framework the question of the reality of the past, even the anti-realist position has reason to reject the application of bivalence only to statements about the past for whose truth or falsity there is presently no available conclusive evidence; for this sort of anti-realist, there is no problem at all in admitting the straightforward truth or falsity of sentences of the sort that Meillassoux considers (for instance statements about the age of the Earth established on the evidentiary basis of radio-carbon dating). More broadly, it is not at all clear how to think about the issue of "anteriority" that forms the linchpin of Meillassoux's argument against the "correlationist" within Dummett's framework or the metaformal one; in particular, if the underlying issue is indeed that of the possibility of a "logical" order of anteriority on the basis of which the position opposed to realism (whether it be called "correlationism" or anti-realism or whatever) seeks to establish logical conditions for the sense or meaningfulness of a class of statements, it is not clear why this "anteriority" should pose any deeper problem than that posed by the "anteriority" of premises to a conclusion in a rational argument, or of a smaller number to a larger one in the sequence of natural numbers.

From the metaformal perspective suggested by Dummett, Meillassoux's "correlationist" does indeed seem, therefore, to be largely a straw man; and his argument against the correlationist, where it does not directly equivocate between ontology and epistemology (as Hallward suggests it does) appears to depend on a closely related failure to consider the implications of semantic considerations for the general realism/anti-realism issue. Does Meillassoux's positive argument for an underlying "hyper-chaos" fare any better? At decisive points in this argument, Meillassoux does appeal directly and decisively to what may seem to be implications of mathematical formalism, and specifically to the implications of the availability of the infinite and transfinite to mathematical thought. For instance, after "disgualifying" the correlationist position on which objects (or our knowledge of them or perhaps their sense) are essentially conditioned by finite forms of human thought, Meillassoux appeals, following Badiou, to Cantor's discovery of the transfinite hierarchy to motivate an anti-"frequentialist" position according to which it is no longer possible to hold natural

or physical laws to be (even in a relative sense) necessary.<sup>25</sup> Meillassoux's basic argument for this conclusion is that since all reasoning about probabilities "presupposes the notion of [a] numerical totality" of possibilities, Cantor's demonstration of the essentially open and non-totalizable hierarchy of infinite sets, if taken as applicable to the question of the conceivability of a total space of possibility, can "provide us with the resources for thinking that the possible is untotalizable" and hence for at least questioning the "necessitarian" assumption that reasoning about the relative probability of laws and events must be possible.<sup>26</sup>

Drawing as it does upon the implications of Cantor's hierarchy of transfinite sets, this argument resembles in some ways the appeal made to formal structures of the infinite in motivating what I have called metaformal realism. However, there are several problems with the appeal as Meillassoux makes it. First, there is in fact no evident direct way to connect Cantor's open hierarchy of the transfinite with any kind of reasoning about probabilities and necessity. As Meillassoux in fact recognizes, it is perfectly possible to determine relative probabilities over domains that admit of infinite or even uncountably infinite ranges of possible values; thus, even if Cantor's results are taken to show that there may be infinitely or even uncountably many "possible worlds," this by itself has no tendency to show that probability measures over the totality of them are not well-defined.<sup>27</sup> More generally, the link between probability and the universe of all sets and quantities which Meillassoux's argument demands here is obscure, and Meillassoux does not clarify how we are to understand it.<sup>28</sup>

But second, and even more problematically, as I argued in *The Politics of Logic*, the availability of

<sup>28</sup> All he says, in fact, is that "...although we have not positively demonstrated that the possible is untotalizable, we have identified an alternative between two options-viz., the possible either does or does not constitute a totalitywith regard to which we have every reason to opt for the second..." Meillassoux, After Finitude, 107. But it cannot be said that the untotalizability of Cantor's hierarchy provides an alternative to a totalizable (or total) possibility space unless we know how to identify the space of possibilities with all of Cantor's hierarchy, and Meillassoux has given us no suggestion as to how to do so; indeed, if we do actually take the "universe" of sets to be untotalizable, this identification (since it calls for identifying all of the possibility space with all of the "universe," which is exactly what does not, on this telling, exist as a whole) is in fact not only unmotivated but in a certain sense impossible.

<sup>&</sup>lt;sup>25</sup> Meillassoux, After Finitude, 100-108.

<sup>&</sup>lt;sup>26</sup> Ibid., 105.

<sup>&</sup>lt;sup>27</sup> Ibid., 102.

the transfinite to thought does not in fact demand the conclusion that Meillassoux follows Badiou in drawing: that of the in-existence of the All of the universe, or the untotalizability of the universe of sets and situations.<sup>29</sup> Rather—and this is the key to what I describe there as another possible post-Cantorian orientation of thought, distinct from and formally opposite to Badiou's own "generic" orientation-it forces a decision on the level of totality and its thinkability. The decision is the one between, on the one hand, the combination of consistency with incompleteness (the alternative Badiou takes and in which Meillassoux apparently follows him) and, on the other, the combination of completeness (or totality) with inconsistency. That is, the implication of Cantor's transfinite and the formal paradoxes and aporias associated with it is not simply to demonstrate or show the incompleteness or inexistence of the Whole, but rather to force the metalogical decision between the two orientations of the generic and the paradoxico-critical, the two orientations that correspond directly, as I have argued above, to the two alternatives of Gödel's disjunctive conclusion.<sup>30</sup>

If Meillassoux had adopted the paradoxico-critical alternative, or even considered it seriously as a possibility for thought, he could by no means have drawn the conclusions that he does about the "necessity of contingency" and the consequent need to assume, outside the "correlationist circle", the absolute existence of an ultimate power of "chaos" by means of which "nothing is or would seem to be, impossible."<sup>31</sup> Rather, on the paradoxico-critical side, he would have had to be driven to consider

<sup>31</sup> Meillassoux, After Finitude, 64.

## IV

Metaformal realism, as I have discussed it here, is an essentially *disjunctive* position, split between affirming the consequences of two quite distinct and mutually incommensurable orientations of post-Cantorian thought, the generic and the paradoxico-critical. As we have seen, Gödel's own disjunctive result witnesses just this disjunction with respect to the powers of human thought in relation to a mathematical reality which the constitutive thought of the infinite determines as the inexhaustible-real: this is, in Gödel's terms, the essential distinction between, on one hand, the assumption of an inherent and transcendent power of human thought to bear witness to consistency by exceeding the powers of any finitely specifiable system of rules, and on the other, an inexhaustible inscription of the undecidable as such, including the undecidability of consistency itself, in the very structure of mathematical reality. Because he was a committed anti-mechanist, Gödel favored the first disjunct (on which the human mind is non-mechanical) and sometimes argued against the tenability of

<sup>&</sup>lt;sup>29</sup> See especially chapters 1 and 9.

<sup>&</sup>lt;sup>30</sup> Meillassoux does recognize that his own conclusions about the bearing of the infinite on the question of chance and law will only be possible if an interpretation of the infinite in terms of inconsistency is first disqualified; thus he argues that, if we are to accept his overarching principle of the "necessity of contingency" we must also hold that "the principle of non-contradiction is absolutely true." Meillassoux, After Finitude, 71. His argument for this, however, is obscure and unconvincing; it proceeds by considering the status of a "contradictory entity;" but even if it is logically coherent to entertain the possibility of such an entity (it is not clear that it is), this possibility has little to do with any consideration that is relevant to establishing the necessity of the law of non-contradiction (i.e., ~(P& ~P) for all statements or propositions P). Again, while Meillassoux does briefly consider (77) the position of paraconsistent logic, he does not actually provide any argument against its applicability to formalize the possibility of real contradictions, or against this possibility itself. For paraconsistency, see, e.g., Graham Priest, In Contradiction: A Study of the Transconsistent (Oxford: Oxford University Press, 1987).

the inherent and structural aporias involved in conceiving of a force of laws and rules that is, within its own sphere, always certainly capable of being complete, but nevertheless always constitutively imbricated with the paradoxes of its own foundation and recurrently involved in the quixotic attempt to prohibit or foreclose its own inherent point of contradiction. On this kind of position, there is no special problem with the coherence of judgments of relative probability or probabilistic causal laws, so long as the general structure of the law as such, as a *consistent* repetition of the same, can be uncritically assumed; but this structure itself always rests on the ultimately aporetic foundation of a consistency that can never be ultimately guaranteed. Since the key point here is not the fixation or absolutization of an unlimited principle of contingency according to which "nothing is...impossible" but rather the acknowledgment of the structurally constitutive possibility of real inconsistency that corresponds to the ultimate unavailability (in accordance with Gödel's second incompleteness theorem) of any intra-systematic guarantees of consistency, this provides another, more critical and less "absolutist," way of considering the nature of scientific (and other) laws and their determination as necessary, one which removes none of the critical force of Hume's problem, but rather situates it within a more radical interrogation of the ultimate basis of the rationally thinkable force of laws as such.

the second on independent grounds, holding both that it ignores the inherent capacity of the human mind to innovate with respect to its guiding axioms and principles and that the existence of absolutely unsolvable problems is untenable since "it would mean that human reason is utterly irrational by asking questions it cannot answer, while asserting emphatically that only reason can answer them."<sup>32</sup>

However, once we have acknowledged the implications of the availability of the infinite to mathematical thought and made the general decision for metaformal realism at all, there are some important senses in which the second disjunct, corresponding to the orientation of paradoxico-criticism, is not only not excluded but also enjoys advantages over the choice for the first disjunct (which Gödel himself preferred). In particular, besides being more obviously compatible with materialism because not in any way at odds with mechanism, the paradoxico-critical outlook makes it possible to preserve an outlook and practice that continues the classical orientation of *criticism* with respect to the capacities and practices of the human subject, in the altered conditions post-Cantorian thought. To gain a sense of these ongoing critical implications, one might usefully juxtapose Gödel's remark about reason asking questions that it cannot answer with the infamous opening lines of Kant's first Critique:

Human reason has this peculiar fate that in one species of its knowledge it is burdened by questions which, as prescribed by the very nature of reason itself, it is not able to ignore, but which, as transcending all its powers, it is also not able to answer.<sup>33</sup>

Kant, of course, was a transcendental idealist; and within the fourfold framework of orientations of thought I developed in The Politics of Logic, Kant's thought remains a paradigm of the pre-Cantorian constructivist (or criteriological) orientation, which is defined by its attempt to assay the boundaries of knowledge from the exterior position of a limit-drawing project committed to saving jointly the ideas of completeness and consistency. In the post-Cantorian context, it is no longer possible to save these ideas jointly, and so the constructivist orientation and <sup>32</sup> Hao Wang, From Mathematics to Philosophy (London: Routledge, 1974), 324, discussed in Feferman "Are there absolutely unsolvable problems? Gödel's dichotomy," 12. The former point, about the non-static nature of the mind, is made against Turing's own position in Kurt Gödel "Some remarks on the undecidability results" in Solomon Feferman, et al., eds., Collected Works, Volume II: Publications 1938-1974, (New York/Oxford: Oxford University Press, 1990), 306. <sup>33</sup> Immanuel Kant, Critique of Pure Reason trans. Norman Kemp Smith (New York: St. Martin's, 1965), A vii.

its associated kind of idealism are both rendered untenable. But by making the paradoxico-critical decision for the combination of a rigorous inquiry into totality with the implication of irreducible paradox at the boundaries, it is possible to maintain the properly critical register of Kant's thought of reflective reason in its ongoing dialectic with itself, and to situate this thought within, as I have argued, a rigorously *realist* position with respect to the relation of thought and being itself. To do so is to transpose the ultimate ground for the development of such a dialectic (now thought more in a properly Platonic rather than a Kantian or Hegelian sense) decisively away from the (pre-Cantorian) Kantian oppositional figure of opposition between the finitude of sensory affection and the absolute-infinite divine intellect capable of intellectual intuition, and to reinvent the possiblities of critique on the ontological real ground of the objective undecidability of problems that are problems for (finite or infinite) thought in itself, given to it at the point of its very contact with the real of being as such.

What, finally, are some of the concrete effects of this transposition for contemporary reflective and critical thought? As I argued in The Politics of Logic, most generally, the necessity, in a post-Cantorian context, of the forced choice between inconsistent completeness and incomplete consistency indicates, as is confirmed by Gödel's development of the philosophical consequences of his own results, that it is impossible by finite, procedural means to confirm rigorously the consistency and completeness of the finitely specifiable procedures of our social-political, practical, and technological worlds. This suggests, as I argued at more length in the book, that it is impossible by finite means to ensure the effectivity of our practices, or procedurally to found whatever faith we may maintain in their ongoing extensibility and capability of continuation. This faith, if it is to be founded at all, must be founded in an essentially infinite capacity of insight and fidelity, bordering on the mystical, to a Real matter of consistency with respect to our own practices that can itself never be guaranteed by any replicable or mechanical procedure; or it must be ceaselessly decomposed and deconstructed at the point of the inherent realism of the problematic and undecidable that is necessarily introduced if this faith cannot be assured at all. Such are the consequences, as I have argued in The Politics of Logic, of the transformative event of the development of formalization in the light of the accessibility of the mathematical infinite that characterizes our time; and such are the stakes, as I have tried to confirm here, of the metaformal realism that this event rigorously motivates and demands.